Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Mathematics-I

Subject Code : 4SC01MAT1 **Branch : B.Sc. (All)**

Semester : 1 **Date : 30/11/2018** **Time : 2:30 To 5:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Find equation of sphere having center $(1,2,3)$ and radius 5. (2)

b) Solve: $y = px + ap(1 - p)$. (2)

c) Check the exactness of the differential equation (2)
$(ax + hy + g)dx + (hx + by + f)dy = 0$.

d) Find order and degree of the differential equation (1)
$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^2 + y = 0$.

e) Find 11th derivative of sin (πx) (2)

f) True/false: every differentiable function has machlaurin's series. (1)

g) Define: Taylor's series expansion of function. (1)

h) Write machlaurin's series of log(1+x). (1)

i) What is polar form of circle having centre at (1 , 1) and radius 4. (2)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find rank of matrix **(5)**

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}.$$

b) Solve $5x - 7y + z = 11, 6x - 8y - z = 15, 3x + 2y - 6z = 7$ using Cremer's method. **(5)**

c) Find Eigen value of **(4)**

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

Q-3 **Attempt all questions** **(14)**

a) Discuss the consistency of the system of equation **(5)**

$$2x + 3y + 4z = 11, x + 5y + 7z = 15, 3x + 11y + 13z = 25.$$

If it is consistent then find it's solution.

b) Find characteristic equation of matrix **(5)**

$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Using it find value of

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

c) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then verify Caley Hamilton's theorem. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Solve: $(x^2 - y^2)dx + 2xy\,dy = 0.$ **(5)**

b) Solve: $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y.$ **(5)**

c) Solve: $\frac{dy}{dx} + \frac{4x}{x^2+1}\, y = \frac{1}{(x^2+1)^3}.$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Find equation of sphere which passes through $(0,0,0), (2,0,0), (0,3,0)$ and $(0,0,4)$. **(5)**

b) Find equation of sphere having end points of diameter are $(1,-2,3)$ and $(0,-1,3)$. **(5)**

c) Write the polar form of the following points : **(4)**

(a) $(1, \sqrt{3})$ (b) $(-\pi\sqrt{2}, \pi\sqrt{2})$

Q-6 **Attempt all questions** **(14)**

a) State and prove Leibnitz's theorem for n^{th} derivative of product. **(6)**

b) Find n^{th} derivative of the following : **(4)**

(a) $\frac{1}{(x-1)(x+2)}$ (b) $\frac{x}{x^2-1}$

c) If $y = \cos(m\sin^{-1}(x))$ then show that $(1-x^2)y_{n+1} - x(2n+1)y_{n+1} + (m^2-n^2)y_n = 0$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) State and prove machlaurin's series of e^x also deduce the machlaurin's series of coshx. **(5)**

b) Find Taylor's series of $x^5 + 4x^4 + 6x^3 - 4x + 1$ at $x = 2$. **(5)**

c) Express $e^{\sin x}$ in powers of x upto x^4 . **(4)**

Q-8 **Attempt all questions** **(14)**

a) State and prove Lagrange's mean value theorem. **(5)**

b) Apply Rolle's theorem for f(x)= (x-1)sinx in the interval [0 , 1] **(5)**

c) State Cauchy's mean value theorem also apply for f(x)= x and g(x) = x+1 in [1,2]. **(4)**